PSTAT 5A: Homework 3

Please note, you are responsible for completing your homework and submitting it to Gradescope. Though the Syllabus allows for the option to not submit homework, it is highly recommended.

1. Consider the random variable X with the following probability mass function (p.m.f.):

k	−2.4	−0.2	0	4
$\mathbb{P}(X = k)$	0.1	0.4	a	0.2

where a is an as-of-yet unknown constant.

   a. What is the value of a?

   b. What is $P(-2.4 \leq X < -0.2)$?

   c. What is $P(X \geq 0)$?

d. If $F_x(x)$ denotes the cumulative distribution function (c.d.f.) of X at x, what is the value of $F_x(0)$?

e. What is $E[X]$?

f. What is $Var(X)$?

2. In a large parking lot, 35% of cars are electric vehicles. A random sample of 200 cars is taken with replacement from this lot, and the number of electric vehicles in the sample is recorded.
   a. Define the random variable of interest and call it X.

   b. Identify the distribution of X, taking care to list out any/all parameter(s)! Also, be sure to check any relevant conditions.

c. What is the probability that this sample contains exactly 72 electric vehicles?

d. What is the probability that between 70 and 73 of the cars in this sample (inclusive on both ends) are electric vehicles?

e. What is the expected number of electric vehicles we expect to observe in this sample?

f. What is the standard deviation of the number of electric vehicles we expect to observe in this sample?

3. The weight of a randomly-selected fish from Lake Gaucho (in bounds) is normally distributed with mean 4 lbs and standard deviation 1.3 lbs. A fish is selected at random from Lake Gaucho and its weight is recorded.
   a. Define the random variable of interest.

b. What is the probability that this fish weighs less than 2 lbs?

c. What is the probability that this fish weighs more than 4.8 lbs?

d. What is the probability that this fish weighs between 2.5 lbs and 3.8 lbs?

e. Suppose, now, that a random sample of 10 fish is caught (assume that the weights of fish in Lake Gaucho are independent), and the number of fish that weight between 2.5 and 3.8 lbs is recorded. What is the probability that exactly 3 of these fish weigh between 2.5 and 3.8 lbs? Hint: you will need to define another random variable.

4. The duration of a flight from SBA to SEA is uniformly distributed between 150 mins and 170 mins. A flight from SBA to SEA is selected at random, and its duration is recorded.
   a. Define the random variable of interest.

b. What is the expected duration of a randomly-selected flight from SBA to SEA?

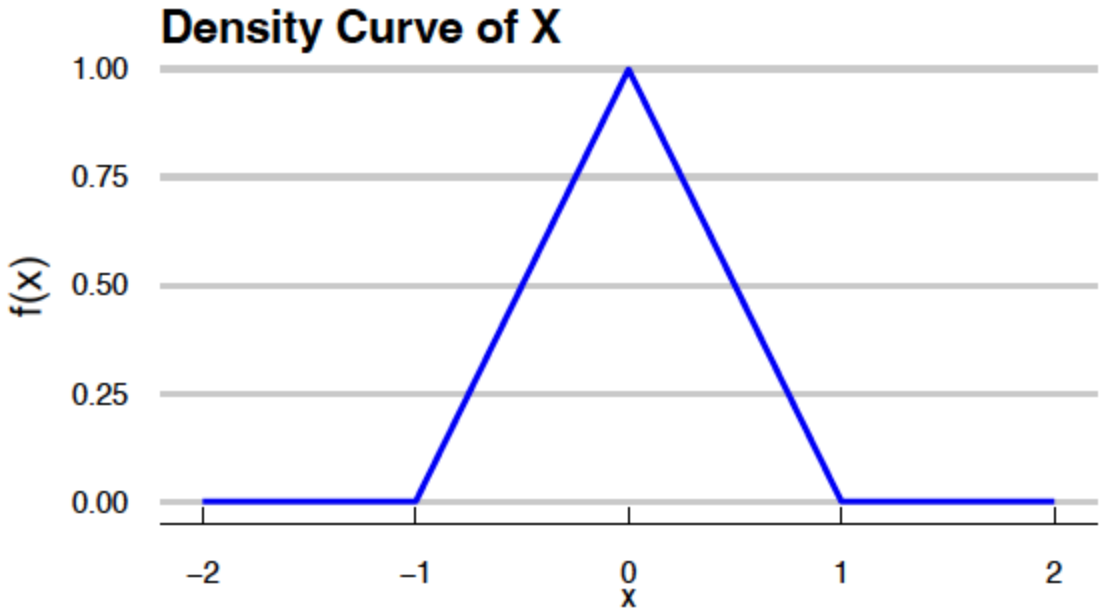
c. What is the standard deviation of the duration of a randomly-selected flight from SBA to SEA?

d. What is the probability that a randomly-selected flight from SBA to SEA will have a duration of 162 mins?

e. What is the probability that a randomly-selected flight from SBA to SEA will have a duration of between 162 and 165 mins?

f. What is the probability that a randomly-selected flight from SBA to SEA will have a duration of between 162 and 180 mins?

5. A random variable X has the following density curve:



a. What is the state space of X?

b. Verify that $f_x(X)$ is a valid probability density function (p.d.f.) Hint: Recall that there are two properties a function $f_x(X)$ must satisfy in order to be a valid p.d.f..

c. What is P(X ≤ −1)?

d. What is P(X ≥ 0)?

e. What is P(−1 ≤ X ≤ −0.5)?

f. What is $P(-1 < X < -0.5)$?

g. What is $P(0.25 \leq X \leq 0.75)$?

6. In each of the parts below, determine whether the provided quantity is a population parameter or a sample statistic. Use this to further determine whether the quantity is a deterministic (i.e. nonrandom) constant, or a random variable.
   a. The proportion of trees in the Amazon rainforest that are mango trees.

   b. The average AQI (Air Quality Index) of 25 randomly-selected cities on August 1, 2022.

   c. The longest amount of time a human can hold their breath underwater.

   d. The median number of words of a sample of 100 randomly-selected books.

7. Suppose that 33% of a particular country's population has a college degree. A representative sample of 243 people is taken, and the proportion of these people who have a college degree is recorded.
   a. Define the parameter of interest, and use the notation discussed in Lecture.

b. Define the random variable of interest; again use proper notation.

c. Check whether the success-failure conditions are satisfied.

d. What is the probability that over 30% of the sample have college degrees?

e. What is the probability that the proportion of people in the sample who have college degrees lies within 5% of the true proportion of 33%?