

PSTAT 5A: Homework 02

Summer Session A 2023, with Ethan P. Marzban

As a reminder, homework is neither collected nor graded. We encourage you to stop by Office Hours to ask any questions you may have about your work, or the problems themselves!

- **1.** A recent survey at a cinema revealed that 80% of moviegoers purchase popcorn and 60% purchase a drink. Additionally, 62.5% of those who purchase popcorn also purchase a drink.
 - (a) Define events, and translate the information provided in the problem. Remember: the events you define should not be conditional.
 - **(b)** What is the probability that a randomly selected moviegoer purchases both popcorn and a drink?
 - (c) What is the probability that a randomly selected moviegoer purchases neither popcorn nor a drink?
- **2.** Consider the experiment of selecting a number at random from the set of positive integers between 1 and 100, inclusive on both ends, and recording the number selected.
 - (a) Write down the outcome space Ω for this experiment.
 - (b) What is the probability that the number selected is even?
 - (c) What is the probability that the number selected is strictly greater than 65?
 - (d) What is the probability that the number selected is even, given that it is strictly greater than 65?
 - (e) If the number is a multiple of three, what is the probability that it is odd?
- **3.** A researcher is interested in the relationship between exercise habits and mental health. To that effect, she surveyed several individuals on their exercise habits as well as their mental health; the results of her survey are displayed in the following contingency table:

1	Mental_Health		
Exercise_Habits	Poor	Fair	Good
Sedentary	30	25	20
Moderately Active	40	35	30
Very Active	45	50	25



A person is selected at random. Use the Classical Approach to Probability wherever necessary.

- (a) What is the probability that the selected person has a sedentary lifestyle?
- (b) What is the probability that the selected person has 'fair' mental health?
- (c) What is the probability that the selected person has both a 'moderately active' lifestyle and 'good' mental health?
- (d) Given that the person has 'good' mental health, what is the probability that they have a 'very active' lifestyle?
- (e) If the person has a 'moderately active' lifestyle, what is the probability that they have 'fair' mental health?
- **4.** Consider events *E* and *F* with $\mathbb{P}(E) = 0.5$, $\mathbb{P}(F) = 0.7$, and $\mathbb{P}(E \cap F) = 0.35$.
 - (a) What is $\mathbb{P}(E \cup F)$?
 - **(b)** What is $\mathbb{P}(E \mid F)$?
 - (c) What is $\mathbb{P}(F \mid E)$?
 - (d) Are *E* and *F* mutually exclusive? Why or why not?
 - (e) Are *E* and *F* independent? Why or why not?

