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## PSTAT 5A: Homework 02

Summer Session A 2023, with Ethan P. Marzban

As a reminder, homework is neither collected nor graded. We encourage you to stop by Office Hours to ask any questions you may have about your work, or the problems themselves!

1. A recent survey at a cinema revealed that $80 \%$ of moviegoers purchase popcorn and $60 \%$ purchase a drink. Additionally, $62.5 \%$ of those who purchase popcorn also purchase a drink.
(a) Define events, and translate the information provided in the problem. Remember: the events you define should not be conditional.
(b) What is the probability that a randomly selected moviegoer purchases both popcorn and a drink?
(c) What is the probability that a randomly selected moviegoer purchases neither popcorn nor a drink?
2. Consider the experiment of selecting a number at random from the set of positive integers between 1 and 100, inclusive on both ends, and recording the number selected.
(a) Write down the outcome space $\Omega$ for this experiment.
(b) What is the probability that the number selected is even?
(c) What is the probability that the number selected is strictly greater than 65 ?
(d) What is the probability that the number selected is even, given that it is strictly greater than 65 ?
(e) If the number is a multiple of three, what is the probability that it is odd?
3. A researcher is interested in the relationship between exercise habits and mental health. To that effect, she surveyed several individuals on their exercise habits as well as their mental health; the results of her survey are displayed in the following contingency table:

|  | Mental_Health |  |  |
| :--- | ---: | ---: | ---: |
| Exercise_Habits | Poor | Fair | Good |
| Sedentary | 30 | 25 | 20 |
| Moderately Active | 40 | 35 | 30 |
| Very Active | 45 | 50 | 25 |

A person is selected at random. Use the Classical Approach to Probability wherever necessary.
(a) What is the probability that the selected person has a sedentary lifestyle?
(b) What is the probability that the selected person has 'fair' mental health?
(c) What is the probability that the selected person has both a 'moderately active' lifestyle and 'good' mental health?
(d) Given that the person has 'good' mental health, what is the probability that they have a 'very active' lifestyle?
(e) If the person has a 'moderately active' lifestyle, what is the probability that they have 'fair' mental health?
4. Consider events $E$ and $F$ with $\mathbb{P}(E)=0.5, \mathbb{P}(F)=0.7$, and $\mathbb{P}(E \cap F)=0.35$.
(a) What is $\mathbb{P}(E \cup F)$ ?
(b) What is $\mathbb{P}(E \mid F)$ ?
(c) What is $\mathbb{P}(F \mid E)$ ?
(d) Are $E$ and $F$ mutually exclusive? Why or why not?
(e) Are $E$ and $F$ independent? Why or why not?

